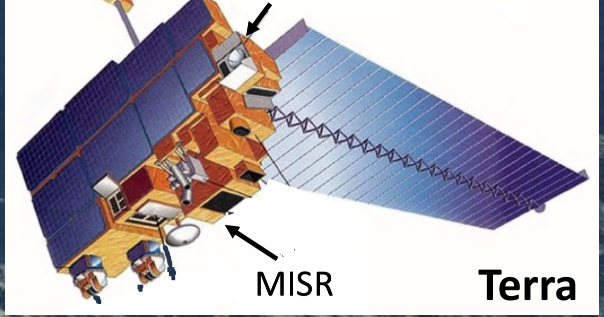
Understanding the Phenomenology of Opaque 3D Cloud Image Formation:

Another Step Toward Cloud Tomography from Space-Based Imaging at Moderate Resolution



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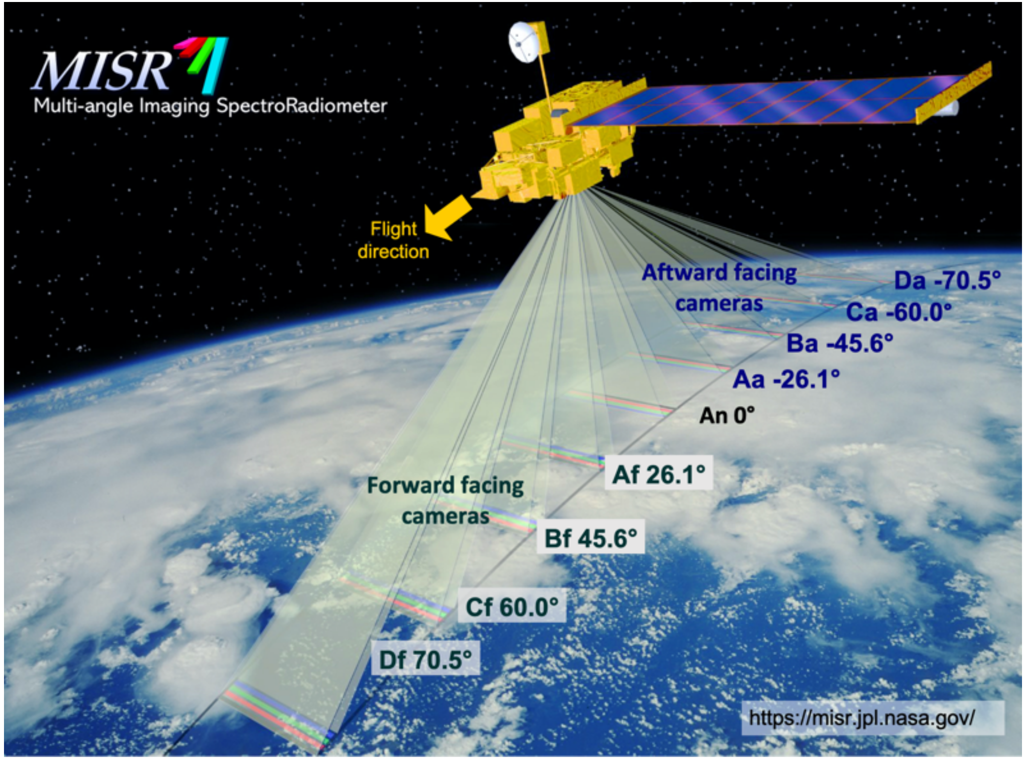
IEEE International Conference on Computational Photography (ICCP) 2021
Physics & Optics Track

Multi-angle Imaging Spectro-Radiometer (MISR) / Terra (launched 1999)



Official cloud products:

- cloud top heights
- height-resolved winds (stereo with time-delay)

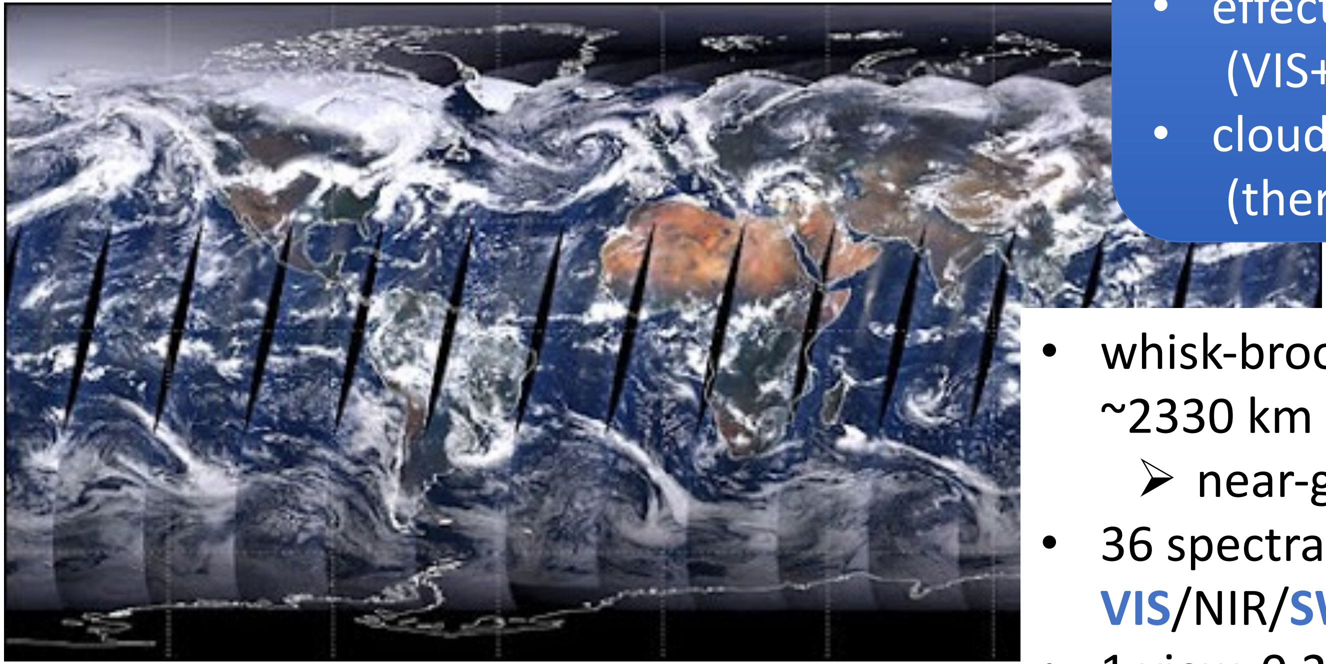
- push-broom acquisition, ~400 km swath
  - ➢ global coverage in 9 days
- 4 spectral channels, all VNIR
- 9 views, 275 m pixels (always for the **red channel** used here)

≈7 minutes from most fore-ward to most aft-word images

MODerate-resolution Imaging Spectrometer (MODIS) / Terra (launched 1999)

Official cloud products:

- cloud optical thickness
- effective particle radius (VIS+SWIR algorithm)
- cloud top height (thermal IR channels)



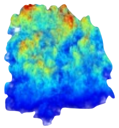
- whisk-broom acquisition, ~2330 km swath
  - ➢ near-global coverage every day
- 36 spectral channels, **VIS**/NIR/**SWIR**/MWIR/LWIR
- 1 view, 0.25–0.50–1.0 km pix s (as wavelength increases)

3D cloud tomography: *Principles*

Multi-angle images
y

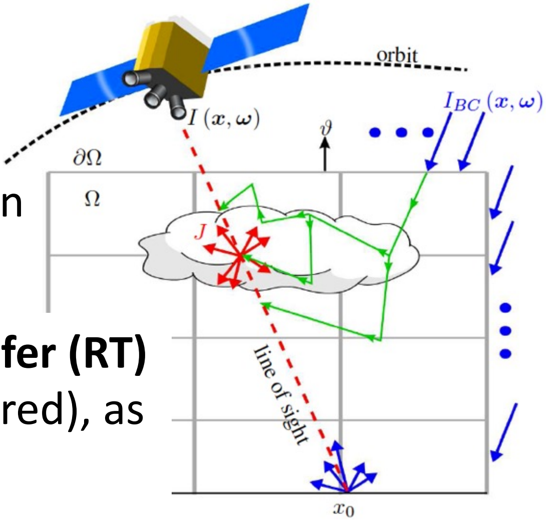
$$\arg\min_{\beta} \|\mathrm{F}(\beta) - \mathrm{y}\|^2$$

Recovered extinction volume $\hat{\beta}$



"β" denotes a 3D gridded field of *unknown* extinction coefficient values.

Need a 3D radiative transfer (RT) solver: SHDOM (restructured), as forward model $\mathrm{F}(\beta)$.



3D RT formulated as two *coupled* integral equations

↓ *Formal* solution of integro-differential 3D RT equation (a.k.a. "upwind sweep")

spatial **integration along beam (x,ω) →**

$$I(x,\omega) = \int J(x',\omega)\beta(x')\,\underbrace{e^{-\int \beta(r)dr}}\,dx' + \underbrace{I_{BC}\underbrace{e^{-\int \beta(x\prime)dx\prime}}_{\text{Beer's law}}}_{\text{solar source } term}$$

directional **integration over incoming ω' →**

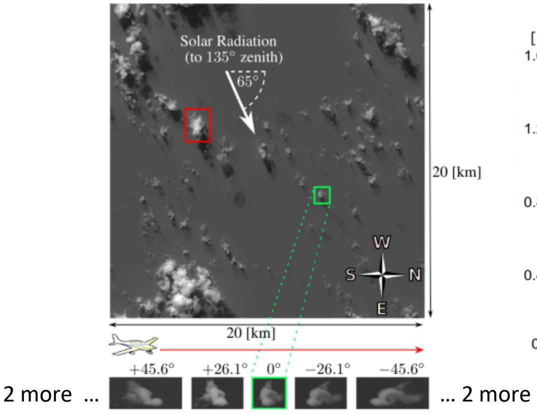
$$J(x,\omega) = \frac{\varpi}{4\pi}\int_{S^2} p(x,\omega\cdot\omega')\,I(x,\omega')\,d\omega'$$

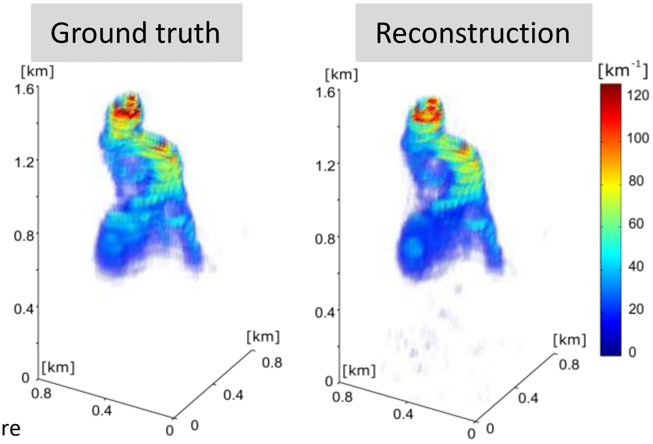
↑ Definition of source *function* $J(x,\omega)$

propagation

scattering

3D cloud tomography: *Demonstration*





→Progress toward CloudCT mission success: poster presentations #12, #16, #17 & #91

- Levis et al. (2015): red channel only, known microphysics (r_e,v_e), 9 views, ***20 m resolution***
  - 46,656 unknowns & 315,018 unknowns, 2-step iteration scheme (1st being linearized) using SHDOM
  - application to data from **Airborne Multi-angle Imaging SpectroPolarimeter (AirMSPI),** ***also 20 m resolution***
- Levis et al. (2017): VNIR multi-spectral
  - basic (profile-only) microphysics (r_e,v_e) ***without*** **SWIR (à la MODIS)** ***nor*** **polarization (à la POLDER)**
- Levis et al. (2020): VNIR multi-spectral/multi-polarimetric
  - potential for a 3D full microphysics (N_e, r_e, v_e) retrieval using **polarization**: [I,Q,U] Stokes vector components

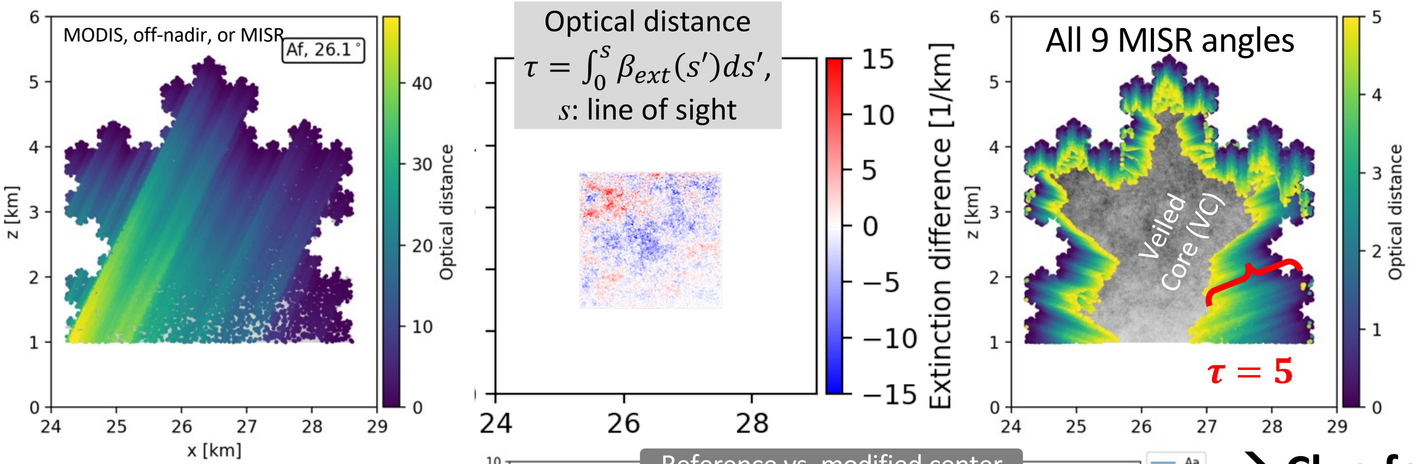
Problem: airborne sensors have ≈20 m pixels

… while space-based ones (MISR + MODIS) have ≈250 & 500 m pixels!

→ *forward 3D RT modeling issues:* voxels can be opaque and/or internally variable

→ *inverse problem solution issues:* larger and more opaque clouds

The "veiled core" of opaque clouds

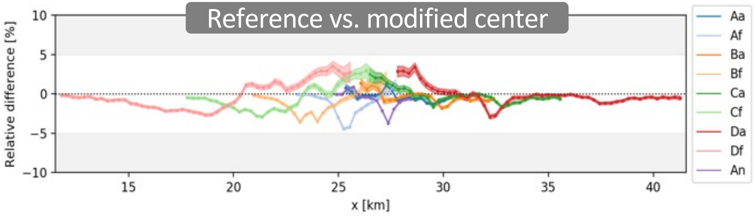


Optical distance

$\tau = \int_0^s \beta_{ext}(s')ds'$,

s: line of sight

$\tau = 5$



Manipulation of extinction field in the VC is *not* detectable in multi-angle imagery (lost in noise).

→ Clue for *inverse problem* form ation: Just one-or-tw unknowns in VC

Problem: airborne sensors have ≈20 m pixels

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→ *forward 3D RT modeling issues:* voxels can be opaque and/or internally variable

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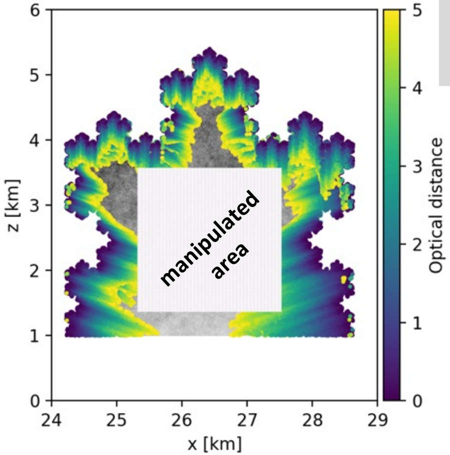
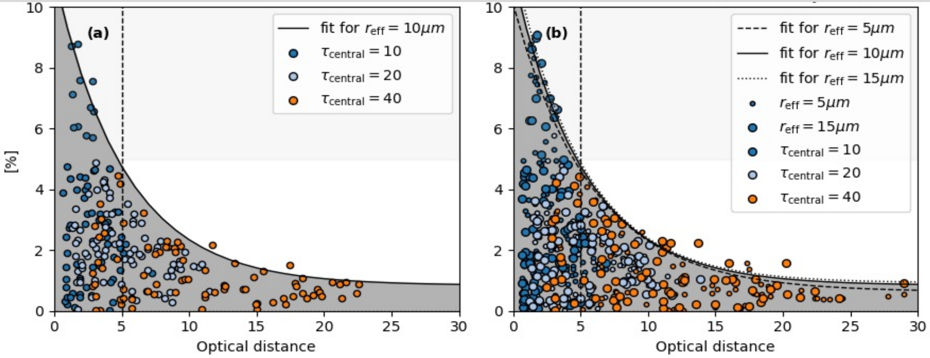
The “veiled core” of opaque clouds

The $\tau \approx 5$ for “VC” threshold for ≈5% tolerance is robust for clouds with sufficient opacity (say, maximum optical thickness in excess of ≈20).





L. Forster, A. B. Davis, B. Mayer, and D. J. Diner (2020), Toward Cloud Tomography from Space using MISR and MODIS: Locating the “Veiled Core” in Opaque Convective 3D Clouds, *J. Atmos. Sci.*, **78**, 155-166 (2021). DOI: https://doi.org/10.1175/JAS-D-19-0262.1

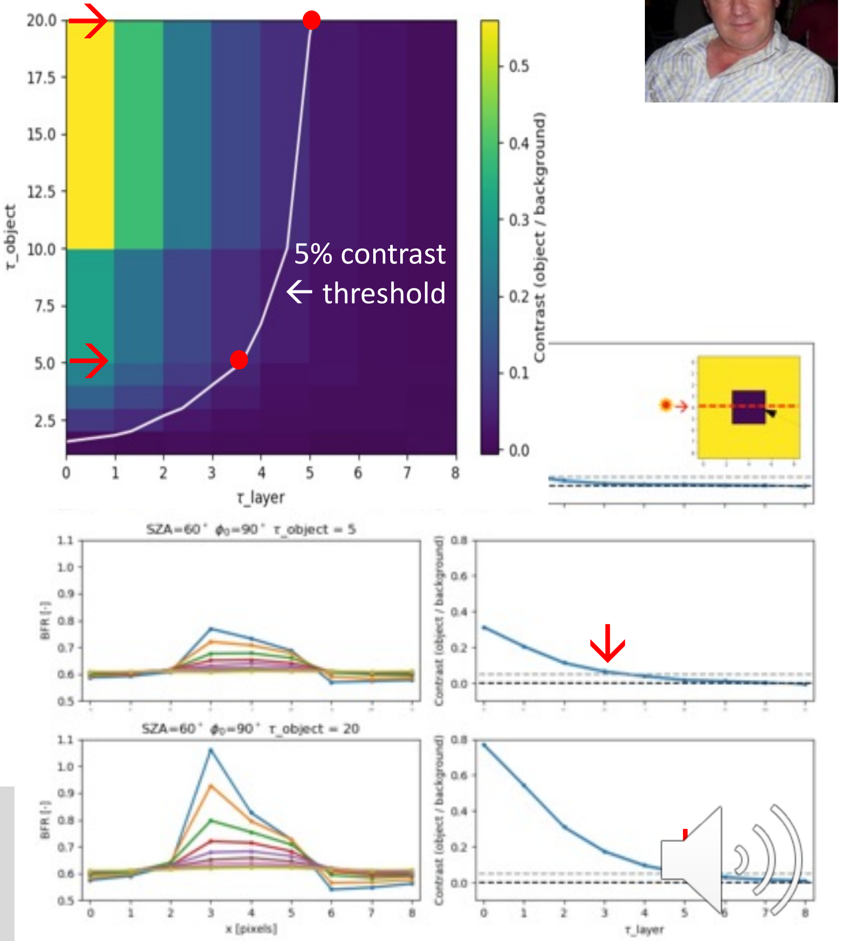
Cloud image formation in VNIR+SWIR:
A tale of two diffusion processes

Diffusion process #1 & #2†

- random walks unfold on the unit sphere (i.e., ***direction*** space)
- in the ***outer shell*** (OS)
  - → along-beam *drift* & lateral *dispersion*
- gradual loss of ***directional*** memory
- *pixel-scale* details in OS matter
- results in identifiable *"**features**"* in cloud imagery

- **RT regime:**
  - ➢ extinction and Beer's law
  - ➢ forward-peaked scattering
  - ➢ small-angle/Fokker-Planck approximation

Superscripts "†" mean "adjoint" or "reciprocal" light, starting at the pixel/direction of interest in the image, propagating back into the cloud, and ending at sources.



5% contrast
← threshold

Cloud image formation in VNIR+SWIR: *A tale of two diffusion processes*

Diffusion process #1 & #2† [or #1 & #3†]

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- **RT regime:**
  - ➢ extinction and Beer's law
  - ➢ forward-peaked scattering
  - ➢ small-angle/Fokker-Planck approximation

Diffusion process [#2]

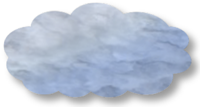
- random walks unfold in 3D ***physical*** space
- in the ***veiled core*** (VC)

- gradual loss of ***positional*** memory
- *cloud-scale* gradients in VC matter
- controls *"**contrast**"* between sunny and shaded sides

- **RT regime:**
  - ➢ scaled/transport extinction
  - ➢ effective isotropic scattering
  - ➢ diffusion/P_1 approximation

A. B. Davis, L. Forster, D. J. Diner, and B. Mayer (2020), Toward Cloud Tomography from Space using MISR and MODIS: The Physics of Image Formation for Opaque Convective Clouds, *J. Atmos. Sci.* (in preparation, preprint at https://arxiv.org/abs/2011.14537).

Cloud image formation in VNIR+SWIR:
A tale of two diffusion processes



Diffusion process #1 & #2† [or #1 & #3†]:

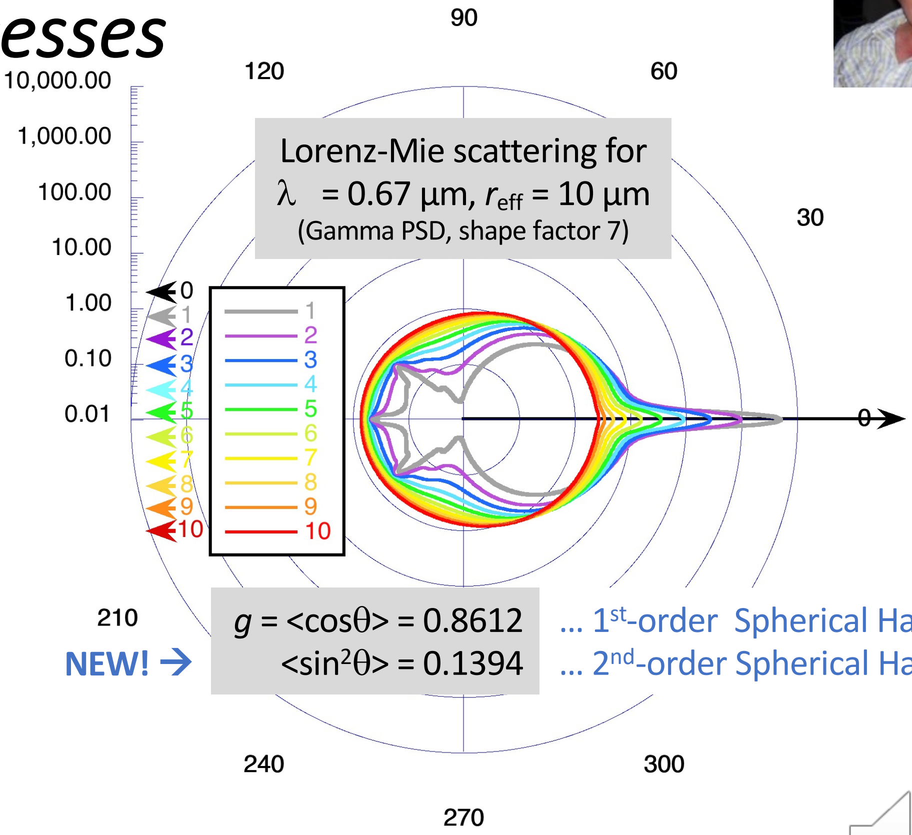
- random walks unfold on 2D sphere (*direction* space)
- in the *outer shell*
- gradual loss of *directional* memory

Characteristic (discrete) time scale to forget solar/sensor direction:

$$N^* = 1/\ln(1/g)$$

$$N^* \approx 6.6 \text{ for } g = 0.86$$

Associated with … $\left\{\begin{array}{l}\textbf{longitudinal drift: } \langle z_{N*}\rangle = \ell(1-g^{N^*+1})/(1-g) \\ \textbf{lateral dispersion: } 2\langle \Delta x^2{}_{N*}\rangle^{1/2}\end{array}\right.$



Lorenz-Mie scattering for
λ = 0.67 µm, r_{eff} = 10 µm
(Gamma PSD, shape factor 7)

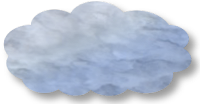
NEW! →

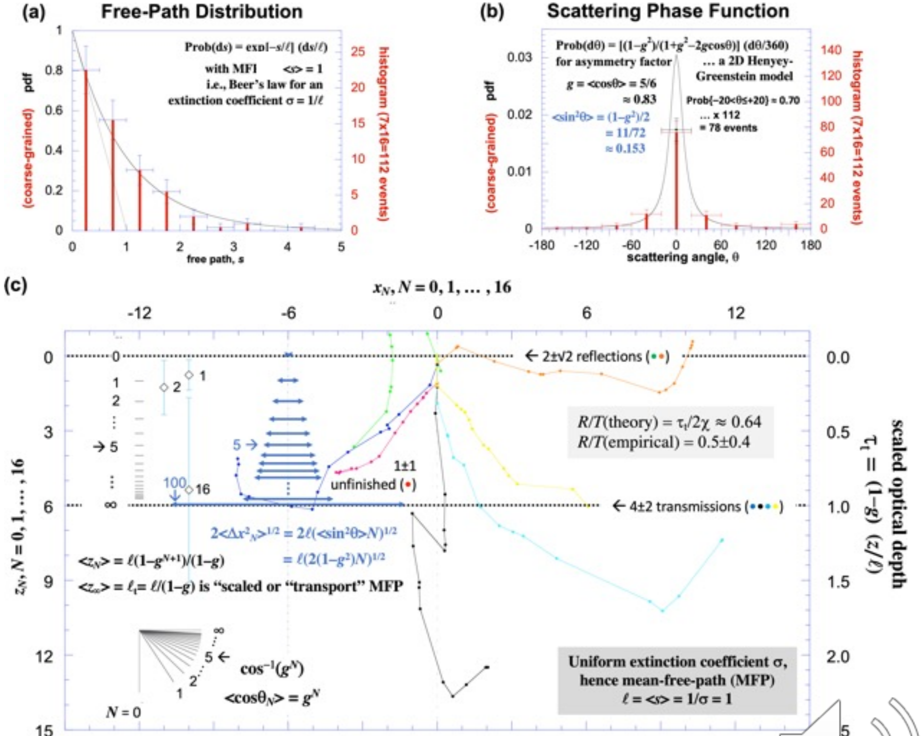
$g = \langle\cos\theta\rangle = 0.8612$ … 1st-order Spherical Harmonic

$\langle\sin^2\theta\rangle = 0.1394$ … 2nd-order Spherical Harmonic

… explains empirical threshold at ≈5 in optical distance that defines the VC.

Cloud image formation in VNIR+SWIR:
A tale of two diffusion processes





Diffusion process #1 & #2† [or #1 & #3†]:

- random walks unfold on 2D sphere (*direction* space)
- in the *outer shell*
- gradual loss of *directional* memory

Characteristic (discrete) time scale to forget solar/sensor direction:

$$N^* = 1/\ln(1/g) \approx (1/g-1)^{-1}$$

$$N^* \approx 5 \text{ for } g = 5/6 \approx 0.83$$

Associated with ... $\begin{cases} \textbf{longitudinal drift: } \langle z_{N^*}\rangle = \ell(1-g^{N^*+1})/(1-g) \\ \textbf{lateral dispersion: } 2\langle \Delta x^2_{N^*}\rangle^{1/2} \end{cases}$

... explains empirical threshold of ≈5 in optical distance that defines the VC.

Cloud image formation in VNIR+SWIR:
A tale of two diffusion processes

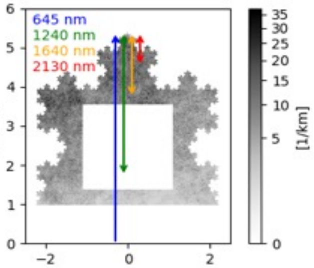
What happens to the now *close-to-isotropic* and already *somewhat-dispersed* forward- or backward-propagating solar radiation when it reaches the veiled core (VC)?

Diffusion process [#2]:

- random walks unfold in 3D *physical* space
- in the *veiled core*
- gradual loss of *positional* memory

Characteristic "diffusion scale," L_d, i.e., the distance from sources where it gets very dark:

$$L_d \times (\text{mean extinction}) = [3(1-\omega)(1-\omega g)]^{-1/2}$$



Let: H_{VC} = bulk size of VC; τ_{VC} = mean optical thickness of VC; and $\langle\rho^2\rangle^{1/2}$ = RMS lateral transport along VC boundary, from entrance to escape. We know that for …

- sensor on ***illuminated* side** [Davis et *al*., 1999ab]

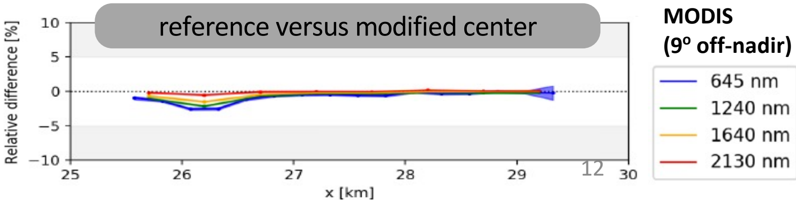
$$\langle\rho^2\rangle^{1/2} \sim H_{VC}/[(1-g)\tau_{VC}]^{1/2}$$

→ **more opaque the VC, less the light will travel;**

- sensor on ***opposite* side** [Davis & Marshak, 2002]

$$\langle\rho^2\rangle^{1/2} \sim H_{VC} \text{ (irrespective of } \tau_{VC} \text{ and } g)$$

→ **light can escape from anywhere.**



→ the VC *grows with orption* across ODIS's SWIR channels

Cloud tomography forward model:

Need high accuracy … *and efficiency!*

Diffusion process #1 & #2† [#3†]:

- random walks unfold on 2D sphere (*direction* space)
- in the *outer shell*
- gradual loss of *directional* memory

➢ ***standard 3D RT equation solver***

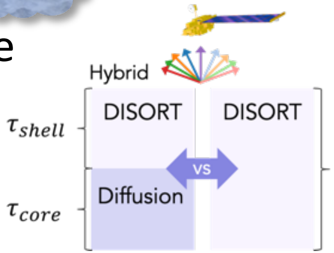
Diffusion process [#2]:

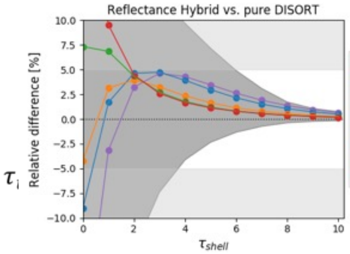
- random walks unfold in 3D *physical* space
- in the *veiled core*
- gradual loss of *positional* memory

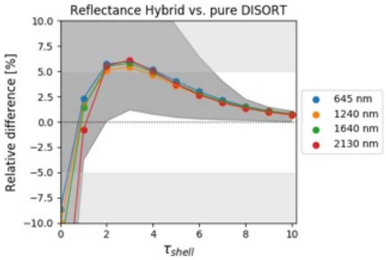
➢ ***efficient diffusion equation solver***

→ Use best of both worlds in a *hybrid* forward 3D RT model.

Hybrid RT: Implementation in 1D

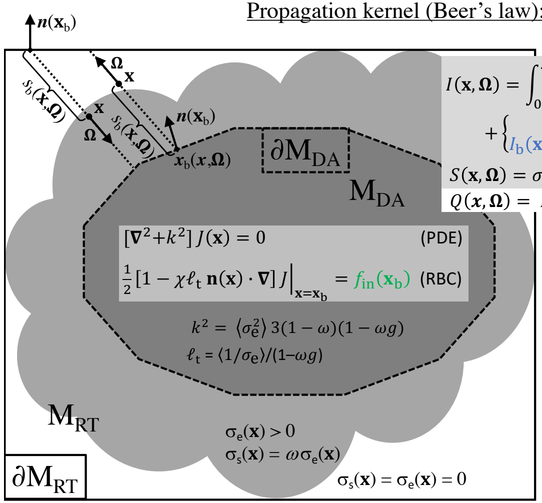






Hybrid RT: Possible Implementation in 3D

Propagation kernel (Beer's law): $T(\mathbf{x},\boldsymbol{\Omega};s) = \exp[-\int_0^s \sigma_\mathrm{e}(\underbrace{\mathbf{x}-\boldsymbol{\Omega}s'}_{\mathbf{x}_{s'}})\mathrm{d}s']$ (BL)



$[\nabla^2+k^2]\,J(\mathbf{x}) = 0$ (PDE)

$\frac{1}{2}[1-\chi\ell_\mathrm{t}\,\mathbf{n}(\mathbf{x})\cdot\nabla]\,J\big|_{\mathbf{x}=\mathbf{x}_\mathrm{b}} = f_\mathrm{in}(\mathbf{x}_\mathrm{b})$ (RBC)

$k^2 = \langle\sigma_\mathrm{e}^2\rangle\,3(1-\omega)(1-\omega g)$

$\ell_\mathrm{t} = \langle 1/\sigma_\mathrm{e}\rangle/(1-\omega g)$

$\sigma_\mathrm{e}(\mathbf{x}) > 0$

$\sigma_\mathrm{s}(\mathbf{x}) = \omega\sigma_\mathrm{e}(\mathbf{x})$

$\sigma_\mathrm{s}(\mathbf{x}) = \sigma_\mathrm{e}(\mathbf{x}) = 0$

$$I(\mathbf{x},\boldsymbol{\Omega}) = \int_0^{s_\mathrm{b}(\boldsymbol{x},\boldsymbol{\Omega})} [S(\mathbf{x}_s,\boldsymbol{\Omega}) + Q(\mathbf{x}_s,\boldsymbol{\Omega})]\,T(\mathbf{x},\boldsymbol{\Omega};s)\,\mathrm{d}s + \begin{Bmatrix} 0 & \text{on } \partial\mathrm{M}_\mathrm{RT} \\ I_\mathrm{b}(\mathbf{x}_\mathrm{b}(\mathbf{x},\boldsymbol{\Omega}),\boldsymbol{\Omega}) & \text{on } \partial\mathrm{M}_\mathrm{DA} \end{Bmatrix} T(\mathbf{x},\boldsymbol{\Omega};s_\mathrm{b}(\mathbf{x},\boldsymbol{\Omega})) \quad \text{(US)}$$

$S(\mathbf{x},\boldsymbol{\Omega}) = \sigma_\mathrm{s}(\mathbf{x})\int_{4\pi} p(\boldsymbol{\Omega}\cdot\boldsymbol{\Omega}')\,I(\mathbf{x},\boldsymbol{\Omega}')\mathrm{d}\boldsymbol{\Omega}'$ (SF)

$Q(\boldsymbol{x},\boldsymbol{\Omega}) = F_0 T(\mathbf{x},\boldsymbol{\Omega}_0;s_\mathrm{b}(\mathbf{x},\boldsymbol{\Omega}_0))\sigma_\mathrm{s}(\mathbf{x})p(\boldsymbol{\Omega}\cdot\boldsymbol{\Omega}_0)$ (ST)

Boundary condition at $\mathbf{x}_\mathrm{b} \in \partial\mathrm{M}_\mathrm{RT}$

$I_\mathrm{b}(\mathbf{x}_\mathrm{b},\boldsymbol{\Omega}) = 0$, for $\mathbf{n}(\mathbf{x}_\mathrm{b})\cdot\boldsymbol{\Omega} < 0$ (HBC)

Two-way coupling at $\mathbf{x}_\mathrm{b} \in \partial\mathrm{M}_\mathrm{DA}$

RT → DA:

$$f_\mathrm{in}(\mathbf{x}_\mathrm{b}) = \int_{\mathbf{n}(\mathbf{x}_\mathrm{b})\cdot\boldsymbol{\Omega}<0} |\mathbf{n}(\mathbf{x}_\mathrm{b})\cdot\boldsymbol{\Omega}|\,I(\mathbf{x}_\mathrm{b},\boldsymbol{\Omega})\mathrm{d}\boldsymbol{\Omega}$$

DA → RT:

$$\begin{cases} I_\mathrm{b}(\mathbf{x}_\mathrm{b},\boldsymbol{\Omega}) = f_\mathrm{out}(\mathbf{x}_\mathrm{b})\dfrac{\mathbf{n}(\mathbf{x}_\mathrm{b})\cdot\boldsymbol{\Omega}}{\pi}, \text{if } \mathbf{n}(\mathbf{x}_\mathrm{b})\cdot\boldsymbol{\Omega} > 0, \\ \text{where } f_\mathrm{out}(\mathbf{x}_\mathrm{b}) = \dfrac{1}{2}[1+\chi\ell_\mathrm{t}\,\mathbf{n}(\mathbf{x})\cdot\nabla]\,J\big|_{\mathbf{x}=\mathbf{x}_\mathrm{b}} \end{cases}$$

Summary & Outlook

- ➢ 3D cloud tomography using multi-angle, multi-spectral, *and multi-pixel* data (i.e., images) collected from current and future **space-based sensors** remains a challenge.
  - Need *adapted* forward model (faster 3D RT solver)
  - Need *informed* inverse problem formulation/solution
    - ❖ **Definition of veiled core (VC) and its outer shell (OS) are key.**
- ➢ Deep dive into the physics of VNIR and SWIR cloud image formation, looking for insights …
  - We uncover *two* complementary diffusion/random-walk processes:

- ❖ First (in OS, near source) and last (in OS, near sensor) are **directional random walks** *on the 2D sphere* that end either in reflection or at the VC, with less and more dispersion, respectively.

  → **pixel-scale "features" → valid targets for detailed cloud tomography**

- ❖ In the VC, solar radiation is transported by a standard **positional random walk** *in 3D space* that ends either in reflection or in transmission, with less and more dispersion, respectively.

  → **cloud-scale *"R/T"* contrast → only 1-or-2 unknowns for the whole VC**

- ➢ This applies to any passive observation of clouds in solar spectrum … naked eyes included!